BBF-003-001616 Seat No. ________

B. Sc. (Sem. VI) Examination

July – 2021

Mathematics : 601 (A)

(Graph Theory & Complex Analysis - II) (Old Course)

Faculty Code : 003
Subject Code : 001616

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

1 Answer all questions : **20×1=20**

(1) Define Simple Graph.

(2) The degree of an isolated vertex is ________

(3) The length of C_n is ________

(4) The number of pendant vertices in a binary tree with 13 vertices is ________

(5) Define cut set.

(6) The number of edge disjoint Hamiltonian circuits in a complete graph K_7 is ________

(7) The edge connectivity of a tree is ________

(8) The chromatic number of a complete graph K_n is ________

(9) Define digraph.

(10) Define planar graph.

(11) Find the radius of convergence of $\sum \frac{z^n}{n-1}$

(12) Define power series.

(13) Write the Maclaurin series for the function $\frac{1}{z+1}$

(14) Find the fixed point of $\frac{6z-9}{z}$

(15) Under the mapping $w = z^2$, the line $x = a$ in $z-$plane maps into a ________ in $w-$plane.

(16) The function $\frac{1}{z}$ has an isolated singularity at ________

(17) What do you mean by removable singular point?

(18) Res $(\tan z, poles) =$ ________

(19) The singular point(s) of $\frac{z+1}{z^3(z+1)}$ is / are ________

(20) Res $\left(\frac{e^z}{z}, 0\right) =$ ________

2 (A) Answer any **three** out of six : **3×2=6**

(1) Find the number of vertices in K_n if it has 45 edges.

(2) Show that a complete graph has always a Hamiltonian circuit.

(3) Prove that the number of vertices in a binary tree is always odd.

(4) In any simple connected planar graph with f regions, n vertices and e edges $(e \geq 2)$ then show that $e \geq \frac{3}{2} f$

(5) Draw the dual graph $K_4^* \, of \, K_4$ Write its n^*, e^* and f^*.

(6) Obtain the incidence matrix of K_4.

(B) Answer any **three** out of six : **3×3=9**

(1) Prove that the number of odd degree vertices in a graph is always even.

(2) Explain Konigsberg bridge problem.

(3) Prove that a tree T with n vertices has $n-1$ edges.

(4) Show that the number of internal vertices in a binary tree is one less than the number of pendant vertices.

(5) Show that every tree with two or more vertices is $2-$chromatic.

(6) Prove that a graph with atleast one edge is $2-$chromatic if and only if it has no odd circuits.

(C) Answer any **two** out of five : **2×5=10**

(1) State and prove characterization of a disconnected graph.

(2) Prove that a simple graph with n vertices and k components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges.

(3) Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even degree.

(4) Prove that a connected planar graph with n vertices and e edges has $e-n+2$ regions.

(5) Prove that $(W_G, \oplus)$ is an abelian group.

3 (A) Answer any **three** out of six : **3×2=6**

(1) Show that for a Mobius mapping, there are atmost two invariant points.

(2) Find the critical point(s) of $w = \frac{z-1}{z+1}$

(3) Show that $e^z = e + e\sum_{n=1}^{\infty} \frac{(z-1)^n}{n!}$

(4) Obtain the Laurent's series of $\frac{1}{z^2-3z+2}$ in $0<|z|<1$

(5) Show that the function exp $\left(\frac{1}{z}\right)$; $0<|z|<\infty$ has an essential singularity at $z=0$.

(6) Find Res $\left(\frac{z^2}{(z-1)(z-2)(z-3)}, 3\right)$

(B) Answer any **three** out of six : **3×3=9**

(1) State and prove Cauchy's residue theorem.

(2) Using Cauchy Residue theorem, evaluate $\int_{|z|=2} \frac{2z+3}{z(z-1)} dz$

(3) Obtain the Laurent's series of $\frac{-1}{(z-1)(z-2)}$ in

(i) $1<|z|<2$ (ii) $2<|z|<\infty$

(4) Prove that $\int_0^\infty \frac{dx}{x^2+1} = \frac{\pi}{2}$

(5) Show that $\cosh\left(z+z^{-1}\right) = \sum_{-\infty}^{\infty} a_n z^n$ where $a_n = \frac{1}{2\pi}\int_0^{2\pi} \cosh(2\cos\theta)\cos n\theta \, d\theta$

(6) Find the bilinear mapping which maps $(-1,\infty,1)$ of $z-$plane into $(2,1,0)$ of $w-$plane,

(C) Answer any **two** out of five : **2×5=10**

(1) State and prove Taylor's series for an analytic function.

(2) Using Cauchy Residue Theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$

(3) Discuss the mapping $w = \frac{1}{z}$

(4) Using Cauchy Residue Theorem, evaluate $\int_{|z|=3} \frac{(3z+2)^2}{z(z-1)(2z+5)} dz$

(5) Show that the set of all bilinear mapping under the composition forms a group.